

Possible Central Extensions of Non-Relativistic Conformal Algebras in 1+1

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Abstract

We investigate possibility of central extension for non-relativistic conformal algebras in 1+1 dimension. Three different forms of charges can be suggested. A trivial charge for temporal part of the algebra exists for all elements of 1-Galilei algebra class. For integer elements of the class we have a charge extension for commutators of temporal and spatial generators. For integer and half integer elements of the class we can have an infinite extension of the generalized mass charge for the Virasoro-like extended algebra. For finite algebras a regular charge inspired by Schrödinger central extension is always possible.

1 Introduction

Central charge plays a crucial role in connection between 2 dimensional critical phenomena and conformal field theory. The classification of the Virasoro algebra representations is solely done on the basis of the value of c , and the universality classes of critical phenomena in 2d are characterized based on the value of c . When looking at non-relativistic conformal algebras, it is not yet clear how decisive the central charge is. But it is clear that it will play an important role. In 2d, unitarity and scale invariance is sufficient to ensure conformal invariance; hence conformal invariance of critical phenomena is assured. Wilson uses renormalization group (RG) [1,2] to make an encompassing theory of Widom [3] and Kadanoff's scaling hypothesis [4]. Using this frame work he managed to explain many features of critical phenomena. Despite this spectacular success of RG, the spectrum of fixed points was not given; it was conformal field theory (CFT) [5] that succeeded in giving at least a partial answer to this question.

Local scale invariance, in 2d can be understood as invariance under any (Anti) holomorphic transformation on the complex plane. The generators of this symmetry form the Witt algebra:

$$[L_m, L_n] = (m - n)L_{m+n}. \quad (1.1)$$

Other commutators are similar for the chiral elements and cross commutators are trivial. However the quantum theory gives rise to an anomaly for the operator product expansion of the energy-momentum tensor, leading to a central charge in the above algebra. Hence the Virasoro algebra appears :

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{C_R}{12}n(n^2 - 1)\delta_{n,-m} \quad (1.2)$$

with a similar change in the chiral component and a separate central charge C_L . Imposing unitarity requires the two central charges to be equal. Interestingly, requiring the representations of the Virasoro algebra representations to be finite dimensional leads to the minimal series. This is a series of CFTs which have :

$$C = 1 - \frac{6}{m(m+1)}, \quad m = 2, 3, \dots \quad (1.3)$$

In this way a whole spectrum of critical points appear [6]. For example $m=3$ or $c=1/2$ corresponds to the critical point of the Ising model [7]. The obvious question is can one do the same for non-relativistic conformal algebras. Interest in non-relativistic conformal algebras has recently grown. They appear as the asymptotic symmetry in AdS/CFT construction [8, 9], mathematicians have paid attention to the mathematical structure of these algebras (See for example [10] and references there in), and in condensed matter physics many applications have been suggested [11,12].

The best known non-relativistic conformal algebra is the Schrödinger algebra which describes symmetries of the Schrödinger equation [13,14]. Beside Galilean invariance this algebra respects scaling invariance:

$$x \rightarrow \lambda x, \quad t \rightarrow \lambda^z t. \quad (1.4)$$

Unlike the standard CFT, here we have anisotropic scaling with a dynamic exponent $z = 2$. Another conformal algebra is the Conformal Galilean Algebra (CGA) which in $2D$ is obtained by a contraction of the conformal algebra [15].

Both Schrödinger algebra and CGA belong to a class of non-relativistic algebras named l-Galilei algebras [16, 17]. These algebras fuse Galilean invariance, causality and scaling in space and time. Each member of this class is characterized by an index "l" related to the dynamic exponent via $z=2/l$. Among these algebras Schrödinger algebra and CGA are most explored [18]- [33]. In Schrödinger algebra, we have an infinite Virasoro like extension which is called Schrödinger-Virasoro algebra [34]. This algebra has a "Mass" central extension which helps to realize the representation theory of the algebra. Besides, it is conserved as a superselection rule in correlation functions. For CGA the story of central charge is interesting. In 1+1 dimensions a satisfactory understanding of this algebra exists [35]. In 2+1 dimensions, there is a known central charge which is called "Exotic" [25]. Physical realization of this charge and symmetry has been of interest. The other elements of l-Galilei algebras are less known¹. Even their representation theory and correlation functions have not yet been worked out. Concerning central charge some progress has been made by Martelli and Tachikawa [38].

In this paper we try to find possible central extensions for these symmetries. This paper is organized as follows. In section 2 we review Schrödinger algebra and its central extension. We then look at CGA in section 3. The core of our paper is in section 4 where we have investigated possible central extension on all element of l-Galilei algebra in 1+1 dimension. We conclude our paper in section 5.

2 Schrödinger Algebra and Mass Central Charge

Schrödinger algebra represents symmetries of Schrödinger equation. Similar to all other non-relativistic conformal algebras, generators of Galilean transformation:

$$P_i = -\partial_i, \quad H = -\partial_t, \quad B = -t\partial_i, \quad J_{ij} = -(x_i\partial_j - x_j\partial_i), \quad (2.1)$$

sit at the center of this algebra. Beside these generators, Schrödinger algebra consists two other generators which are dilation

$$D = -(t\partial_t + \frac{1}{2}x_i\partial_i), \quad (2.2)$$

and Special Schrödinger Transformation (SCT):

$$K = -(tx_i\partial_i + t^2\partial_t). \quad (2.3)$$

These generators produce transformations as:

$$\vec{x} \rightarrow \frac{\mathcal{R}\vec{x} + \vec{v}t^2 + \vec{b}}{\gamma t + \delta}, \quad t \rightarrow \frac{\alpha t + \beta}{\gamma t + \delta} \quad (2.4)$$

¹For a discussion on physical significance of elements with $l = 4$ and $l = 6$ see [36, 37]

in which $\mathcal{R} \in SO(D)$ and $\alpha\delta - \beta\gamma = 1$. The algebra is still incomplete to thoroughly represent symmetries of Schrödinger equation. Consider a plane wave $\psi_{E,\vec{p}} = a \exp(iEt - i\vec{p} \cdot \vec{x})$. Now, if we define the action of translation on ψ as:

$$T_{P(\vec{b})}\psi(\vec{x}) = \psi(P(-\vec{b})(\vec{x}, t)), \quad (2.5)$$

we observe that as our expectation the result is another solution of Schrödinger equation. For the action of boost however we face problem. For a boost \vec{v} we expect

$$T_{B(\vec{v})}\psi(\vec{x}) = \psi(B(-\vec{v})(\vec{x}, t)). \quad (2.6)$$

The new ψ however is not a solution of Schrödinger equation. To be a solution we can define the action of boost on ψ as:

$$T_{B(\vec{v})}\psi(\vec{x}) = \exp(-im\vec{v} \cdot \vec{x} + \frac{i}{2}m\vec{v}^2 t)\psi(B(-\vec{v})(\vec{x}, t)). \quad (2.7)$$

The new ψ is another solution of Schrödinger equation and as we expect represents a plane wave with new energy and momentum in boosted coordinate set:

$$\vec{P} \rightarrow \vec{P} + m\vec{v}, \quad E \rightarrow \frac{(\vec{P} + m\vec{v})^2}{2m}. \quad (2.8)$$

Now we face a new problem and according to transformation (2.7), $T_{P(b)}$ and $T_{B(\vec{v})}$ do not commute and we have:

$$T_{P(b)}T_{B(\vec{v})} = T_{P(b)}T_{B(\vec{v})} \exp(im\vec{v} \cdot \vec{b}). \quad (2.9)$$

So, to realize a proper transformation of solutions of Schrödinger equation we find that boost and translation in space should not commute and as the simplest form we have a central charge

$$[B_i, P_j] = M\delta_{ij}. \quad (2.10)$$

M is nothing but the mass central extension of Schrödinger algebra. To realize this central charge as a generator we may utilize a new coordinate ξ and define a new field ϕ as a Fourier transform of ψ with respect to mass [32]:

$$\psi(\vec{x}, t) = \frac{1}{\sqrt{2\pi}} \int_R d\xi e^{-im\xi} \phi(\xi, \vec{x}, t) \quad (2.11)$$

Now, Schrödinger equation for ψ in $\mathbb{R}^{d,1}$ space turns to Klein-Gordon equation for ϕ in $\mathbb{R}^{d+1,1}$ space:

$$2\frac{\partial^2 \phi}{\partial t \partial \xi} + \nabla^2 \phi = 0. \quad (2.12)$$

Metric for this space is:

$$ds^2 = 2dtd\xi + \vec{dx} \cdot \vec{dx} \quad (2.13)$$

Under boost transformation ξ now transforms as:

$$\xi \rightarrow \xi + \vec{v} \cdot \vec{x} - \frac{1}{2} \vec{v} \cdot \vec{v} t. \quad (2.14)$$

The action of Sch_1 on $\phi(\xi, x, t)$ now reads as:

$$\begin{aligned} B &= -t\partial_x + x\partial_\xi, & M &= \partial_\xi, \\ D &= -2t\partial_t - x\partial_x + \Delta, & K &= -2t^2\partial_t - 2tx\partial_x + x^2\partial_\xi - 2\Delta t. \end{aligned} \quad (2.15)$$

in which Δ is scaling weight. The set of operators H, D and K form an $SL(2, R)$ algebra. So, the algebra can be extended in this direction to form Schrödinger-Virasoro algebra [34]. The central charge then needs to be extended as well and then we have a new set of operators for M. The infinite extended algebra then can be realized as:

$$\begin{aligned} T^n &= -t^{n+1}\partial_t - \frac{(n+1)}{2}t^n x\partial_x - \frac{(n+1)}{2}\Delta t^n + \frac{n(n+1)}{4}t^{n-1}x^2\partial_\xi \\ P^q &= -t^{q+1}\partial_x + (q+1)t^q x\partial_\xi \\ M^n &= t^n\partial_\xi \end{aligned} \quad (2.16)$$

in which $n \in \mathbb{Z}$ and $q \in \mathbb{Z} + \frac{1}{2}$. The non-vanishing commutators of these generators are:

$$\begin{aligned} [T^m, T^n] &= (m-n)T^{m+n} & [T^m, P^q] &= \left(\frac{m}{2} - q\right)P^{m+q} \\ [T^m, M^n] &= -nM^{m+n}, & [P^p, P^q] &= (p-q)M^{p+q} \end{aligned} \quad (2.17)$$

Now consider a scaling field with a weight "h". Now, other operators play the ladder role. Inspired by definitions in CFT one may define primary fields as those to be annihilated by T^n, P^q, M^n $n > 0$. Now, other operators play ladder role:

$$\begin{aligned} T^{(-n)}|h\rangle_{\mathcal{M}} &\rightarrow |h+n\rangle_{\mathcal{M}} \\ P^{(-q)}|h\rangle_{\mathcal{M}} &\rightarrow |h+q\rangle_{\mathcal{M}} \\ M^{(-n)}|h\rangle_{\mathcal{M}} &\rightarrow |h+n\rangle_{\mathcal{M}} \end{aligned} \quad (2.18)$$

Sine M^0 is the central charge, the index \mathcal{M} is not changed within an irreducible representation. Now, one can look for descendant fields, Kac determinant, Verma modules, null vectors or Jordan representation.

3 Conformal Galilean Algebra

Conformal Galilean algebra (CGA) was found in [15] and then independently was rediscovered in [16]. In d dimension this algebra can be obtained from contraction of conformal algebra. In contraction we let:

$$x \rightarrow x/c \quad t \rightarrow t \quad c \rightarrow \infty \quad (3.1)$$

Under such contraction, Poincare transformations reduces to Galilean transformation and conformal symmetry reduces to CGA. Physically we expect it means investigating a system under conformal symmetry in low speed or energy. The algebra consists of:

$$\begin{aligned} P_i &= -\partial_{x_i}, & K_i &= -t\partial_{x_i}, & F_i &= -t^2\partial_{x_i}, \\ H &= -\partial_t, & D &= -(t\partial_t + x\partial_x), & C &= -(2tx\partial_x + t^2\partial_t), \\ J_{ij} &= -x_i\partial_{x_j} + x_j\partial_{x_i}. \end{aligned} \quad (3.2)$$

In 2+1 dimensions the algebra admits a central charge Ξ [25]

$$[K_i, K_j] = \Xi\epsilon_{ij}, \quad [P_i, F_j] = -2\Xi\epsilon_{ij} \quad ; \quad i, j = 1, 2, \quad (3.3)$$

which is called exotic in the literature. The physical significance of this charge and its representation theory has been of interest [27]- [28].

Dilations operator "D" and two other operators (C and H) obey $SL(2, \mathbb{R})$ commutation relations. The algebra can be extended in this direction and similar to the Schrödinger one we have a Virasoro like extension of CGA which is called full CGA in the literature:

$$\begin{aligned} T^n &= -t^{n+1}\partial_t - (n+1)t^n x^i \partial_{x_i}, \\ P_i^n &= -t^{n+1}\partial_{x_i} \\ J_{ij} &= -x_i\partial_{x_j} + x_j\partial_{x_i}. \end{aligned} \quad (3.4)$$

In 1+1 we can obtain this algebra from contracting CFT_2 generators [21]. Consider elements of Virasoro algebra in complex plane (1.2). Now, under contraction (3.1) and redefining operators as:

$$\begin{aligned} T^n &= L^n + \bar{L}^n \\ P^n &= \frac{1}{c}(L^n - \bar{L}^n) \end{aligned} \quad (3.5)$$

we end up with full CGA in (3.4). Now, consider central charges of CFT_2 in (1.2). Under contraction limit we are left with new charges for full CGA [11, 31]:

$$\begin{aligned} [T^m, T^n] &= (m-n)T^{m+n} + \frac{1}{12}C_T m(m^2-1)\delta_{m,-n}, \\ [P^m, P^n] &= 0, \\ [T^m, P^n] &= (m-n)T^{m+n} + \frac{1}{12}C_P m(m^2-1)\delta_{m,-n}, \end{aligned} \quad (3.6)$$

Through this contraction L_0 and \bar{L}_0 go to T^0 and P^0 . Now if we suppose that as the generators, representations of full CGA can be obtained from contracting the representations of CFT_2 too, then we are done with quantizing CGA. Generally when an algebra is contracted there is no necessity that its representations as well can be contracted to yield representations of the obtained algebra. For CGA however, this is the case. To find justifications in this regard see [35]. Under such procedure, representations with charges as C_T and C_P in CGA are obtained from representations of CFT with charges equal to:

$$\begin{aligned} C_T &= \frac{C_R + C_L}{2}, \\ C_P &= \frac{C_L - C_R}{2c}. \end{aligned} \quad (3.7)$$

All other interesting elements such as null vector, Kac determinant etc. can be inherited from CFT_2 through contraction [35].

4 The general class of l -Galilei Algebra

Schrödinger algebra and CGA belong to a general class of non-relativistic algebras named l -Galilei algebra. This algebra has been derived from a phenomenological approach by Henkel [16] and from algebraical view by Negro et al. [17]. If we look for an algebra which let global conformal transformation of time, an scale invariance in space and beside respects Galilean casualty we end up with an algebra produced by generators:

$$\begin{aligned} H &= -\partial_t, & C &= -(2ltx\partial_x + t^2\partial_t), & D &= -(t\partial_t + lx\partial_x) \\ P_i^q &= (-t)^q\partial_{x_i}, & J_{ij} &= -x_i\partial_{x_j} + x_j\partial_{x_i}. \end{aligned} \quad (4.1)$$

where $l \in \frac{\mathbb{N}}{2}$ and $q = 0, \dots, 2l$. These operators produce transformations as:

$$\vec{x} \rightarrow \frac{\mathcal{R}\vec{x} + t^{2l}\vec{c}_{2l} + \dots + t\vec{c}_1 + \vec{c}_0}{(\gamma t + \delta)^{2l}} \quad t \rightarrow \frac{\alpha t + \beta}{\gamma t + \delta} \quad (4.2)$$

where $\mathcal{R} \in SO(d)$, $\vec{c}_n \in \mathbb{R}$ and $\alpha\delta - \beta\gamma = 1$. Dilation operator produces an anisotropic rescaling of space and time:

$$\vec{x} \rightarrow \lambda x \quad t \rightarrow \lambda^z t, \quad (4.3)$$

where $z = \frac{1}{l}$. Schrödinger algebra is the first element of this class or $l = \frac{1}{2}$. The second element $l = 1$ is nothing but CGA. Other elements are not well-known as their former elements. Non-zero commutators of the algebra reads as:

$$\begin{aligned} [D, H] &= H, & [D, C] &= -C, & [C, H] &= 2D, \\ [J_{ij}, P_k^q] &= \delta_{ik}P_j^q - \delta_{jk}P_i^q & [H, P_i^q] &= -qP_i^{q-1}, & [D, P_i^q] &= (l - q)P_i^q, \\ [C, P_i^q] &= (2l - q)P_i^{q+1}. & [J_{ij}, J_{kl}] &= \delta_{ik}J_{jl} + \delta_{jl}J_{ik} - \delta_{il}J_{jk} - \delta_{jk}J_{il}. \end{aligned} \quad (4.4)$$

In all elements of this class of symmetries still H, D and C form an $SL(2, R)$ algebra. So, we can have a Virasoro-like infinite extension in this direction:

$$\begin{aligned} T^n &= -t^{n+1}\partial_t - l(n+1)t^n x_i \partial_i, \\ P_i^q &= -t^{q+l}\partial_i, \\ J_{ij}^n &= -t^n(x_i\partial_j - x_j\partial_i) \end{aligned} \quad (4.5)$$

in which $n \in \mathbb{Z}$ and $q \in \mathbb{Z} + l$. Since P^q act basically on space we have called them "spatial operators" in this note. To distinguish T^n generators we have called them *temporal* ones. These generators produce the following algebra:

$$\begin{aligned} [T^m, T^n] &= (m - n)T^{m+n}, & [T^m, J_{ij}^n] &= -nJ_{ij}^m, \\ [T^m, P_i^q] &= (lm - q)P_i^{m+q}, & [J_{ij}^m, P_k^q] &= \delta_{ik}P_j^{m+q} - \delta_{jk}P_i^{m+q}, \\ [J_{ij}^m, J_{kl}^n] &= \delta_{ik}J_{jl}^{m+n} + \delta_{jl}J_{ik}^{m+n} - \delta_{il}J_{jk}^{m+n} - \delta_{jk}J_{il}^{m+n}, \end{aligned} \quad (4.6)$$

Note that the finite subalgebra (4.4) can be reached via elements with $m = 0, \pm 1$ and $q = -l, -l + 1, \dots, l - 1, l$ in (4.5).

Investigating central extension for these algebras, it has been suggested that only two forms of central extension can exist [38]. For any d and half integer l we can have a charge as

$$[P_i^p, P_j^q] = I^{pq} \delta_{ij} M, \quad (4.7)$$

in which I^{pq} is an antisymmetric tensor. This is an extension of the mass charge for all half integer elements of l-Galilei algebra. For $d = 2$ and integer l we can have an extension of Exotic charge:

$$[P_i^p, P_i^q] = I^{pq} \epsilon_{ij} \Xi. \quad (4.8)$$

Though these charges have been suggested as the most general case for all elements of l-Galilei algebra, if we look at the case of CGA we find that other forms of the central charges can exist in 1+1. Now, we follow on to check if we can find different charges for these algebras in 1+1. Concerning the central charge basically we can have three forms of central extension:

1: An extension in which the charge is outcome of commutators of temporal operators which we call T charge.

2: Another extension that let the charge results from between temporal and spatial operators which we have called a B charge in this note.

3: A charge that can be an outcome of commutators of spatial operators. Since Schrödinger algebra's mass central charge is of this kind, we have called them M charge.

The T charge trivially exist for any element of the l-Galilei class. It roots from the fact that for these algebras the temporal generators always form a $SL(2, R)$ subalgebra. So, we have:

$$[T^m, T^n] = (m - n) T^{m+n} + \frac{1}{12} C_T m(m^2 - 1) \delta_{m+n, 0}. \quad (4.9)$$

Now, let's follow on to investigate if a charge can be obtained through commutation of T^m and P^q for arbitrary l . We have two forms of l : half integer l and integer l .

Case 1a: Integer l for B central charge

We should consider that we have such charge for CGA (3.6) which belongs to $l = 1$ of l-Galilei class. Now, we investigate if such extension can be possible for other algebras of l-Galilei class. We desire the central charge obeys the equation:

$$[T^m, P^n] = (lm - n) P^{m+n} + C_B f(m) \delta_{m+n, 0}. \quad (4.10)$$

Now, let's write Jacobi identity:

$$[[T^m, T^n], P^q] + [[T^n, P^q], T^m] + [[P^q, T^m], T^n] = 0. \quad (4.11)$$

It leads to the following equation for the central charge:

$$[(m-n)f(m+n) + (q-ln)f(m) + (lm-q)f(n)]C_B\delta_{m+n+q,0} = 0, \quad (4.12)$$

and thereby:

$$(m-n)f(m+n) - ((l+1)n+m)f(m) + ((l+1)m+n)f(n) = 0. \quad (4.13)$$

Now, one can easily check that if we suppose f is a polynomial function then we will have two choices. For $l = 1$ we have CGA choice or

$$f(m) = am + bm^3. \quad (4.14)$$

For other integer l we only have a choice as $f(m) = am$. So, for $l > 1$ we are left with the following charge:

$$[T^m, P^q] = (lm-q)P^{m+q} + C_B m \delta_{m+q,0}. \quad (4.15)$$

Case 1b: Half integer l for B central charge

While temporal operators are indexed by integer numbers, spatial operators are indexed by half integer numbers. So, one may seek a central charge as

$$[T^m, P^q] = (lm-n)P^{m+q} + C_B f(m)\delta_{m+2zq,0}, \quad (4.16)$$

in which z is an integer number. Now, let's see if Jacobi identity are satisfied by this equation. We desire

$$[[T^m, T^n], P^q] + [[T^n, P^q], T^m] + [[P^q, T^m], T^n] = 0, \quad (4.17)$$

which reduces to:

$$(m-n)f(m+n)\delta_{(m+n+2qz),0} + (q-ln)f(m)\delta_{m+2zn+2zq,0} + (lm-q)f(m+q)\delta_{n+2zq+2zm,0} = 0. \quad (4.18)$$

Now, we choose $m = -2qz - n$ and thereby we have:

$$(2qz-2n)f(-2qz) + (q-ln)f(-2qz-n)\delta_{n(2z-1),0} - (l(2qz+n)-q)f(-2qz-n+q)\delta_{n+2z(q-2qz-n),0} = 0. \quad (4.19)$$

This equation should hold for any "n". So, we have no choice but $f(-2qz) = 0$. Thereby any form of central charge as equation (4.16) is impossible.

Now, we follow to investigate if the algebras admit central extension for commutators of spatial operators. Inspired by Schrödinger *mass* central extension we have called such extension as M charge in this note.

Case 2a: M Charge for infinite-extended l-Galilei algebra

Schrödinger algebra has a M charge for commutators of its spatial operators B and P (2.10). In the compact form of equation (4.5) for this charge we have:

$$[P^{\frac{1}{2}}, P^{-\frac{1}{2}}] = M. \quad (4.20)$$

One may expect an extension as

$$[P^q, P^r] = M f(q) \delta_{q+r,0} \quad (4.21)$$

for infinite extended of the algebra. The Schrödinger-Virasoro however has a different form of M charge in (2.17). Instead of having only one operator to embed mass charge to infinite extension of Schrödinger algebra we need to define an infinite set of operators (2.16). To observe that a central extension as of (2.16) can't hold for any element of the l-Galilei class we write Jacobbi identity for it:

$$[[T^m, P^q], P^r] + [[P^q, P^r], T^m] + [[P^r, T^m], P^q] = 0. \quad (4.22)$$

Plugging M charge from (4.21) leads to equation:

$$f(m+q) = -\frac{(lm+m+q)}{(lm-q)} f(q). \quad (4.23)$$

Now, we try to interpret $f(q+2)$ in terms of $f(q)$. In one hand we have:

$$f(q+2) = \frac{l+1+(q+1)}{q+1-l} f(q+1) = \frac{l+1+(q+1)}{q+1-l} \frac{l+1+q}{q-l} f(q) \quad (4.24)$$

and in the other hand we have:

$$f(q+2) = \frac{2l+2+q}{q-2l} f(q). \quad (4.25)$$

So, we are left with the following constraint:

$$\frac{l+1+(q+1)}{q+1-l} \frac{l+1+q}{q-l} = \frac{2l+2+q}{q-2l}, \quad (4.26)$$

which trivially can't be held by different values of q . So, any extension as of (4.21) for infinite extended algebra is impossible. The only possibility is to have an infinite set of generators. Inspired by Schrödinger-Virasoro algebra we can write such charge as:

$$\begin{aligned} [T^m, P^q] &= (lm-q)P^{m+q} \\ [T^m, M^n] &= -nM^{m+n}, \\ [P^p, P^q] &= (p-q)M^{p+q}. \end{aligned} \quad (4.27)$$

Case 2b: M Charge for finite l-Galilei algebra

We observed that for infinite extended algebra the constraint of Eq. (4.26) can't be held and a central charges as of Eq. (4.21) is impossible. For finite algebra however we always can save this charge from Jacobi identity constraint. For finite algebra in Eq. (4.23), "m" can only take the values of 0 and ± 1 . So, $f(q+1)$ can be expressed in terms of $f(q)$ and since q ranges from $-l$ to l , we find that the central charge needs to be expressed as:

$$[P^{-l+i}, P^r] = M(-1)^i \frac{i!(2l-i)!}{(2l)!} \delta_{-l+i+r,0}. \quad (4.28)$$

For $l = \frac{1}{2}$ or Schrödinger algebra this charge is nothing but the famous mass charge.

5 Concluding Remarks

Despite conformal symmetries which are recognized as symmetries to describe critical phenomena at equilibrium, the non-relativistic ones have been suggested to describe either dynamics of critical phenomena in some regimes or strongly anisotropic systems (see for example [7, 34] and references therein). In this regard the dynamical index $z = \frac{1}{l}$ expresses either "How space is different from time" or "How two directions are anisotropic in space". Many models have been suggested for Schrödinger algebra ([34]) which is identified by $l = \frac{1}{2}$. Two-point correlators identified by $l = 4$ symmetry have been suggested to find application in describing a certain multicritical point called "Lifshitz point" in [16]. As well application of this symmetry has been suggested for ANNNI model in [36]. For discussion on $l = 6$ see [7].

In our work we tried to go one step ahead to know l-Galilei algebras. The detailed of our discussion have been summarized in tables 1.

1. The T charge trivially exists for any element of the class.
2. A form of central extension as of B charge only holds for integer- l elements of the class.
3. A regular central extension as of M charge is impossible for infinite-extended algebras. An extended form of mass charge used in Schrödinger-Virasoro algebra however is possible for all elements of the class.
4. Though a regular extension for mass charge is impossible for infinite-extension of any element of the class, it is however possible for finite algebras.

We investigated possibilities for central extension of all elements of l-Galilei class in $1+1$. The case of $1+1$ is important since CFT, Schrödinger-Virasoro algebra and CGA have found solution there. So, there is more chance that we know other elements of the class in this dimension.

	Algebra		Eq. #	
T charge	half-integer l	✓	(4.9)	$[T^m, T^n] = (m - n)T^{m+n} + \frac{1}{12}C_T m(m^2 - 1)\delta_{m+n,0}$
	integer l	✓	(4.9)	$[T^m, T^n] = (m - n)T^{m+n} + \frac{1}{12}C_T m(m^2 - 1)\delta_{m+n,0}$
B charge	half-integer l	×	—	
	integer l	✓	(4.15)	$[T^m, P^q] = (lm - q)P^{m+q} + C_B m\delta_{m+q,0}$
Generalized M charge	infinite extended l-Galilei	✓	(4.27)	$[P^p, P^q] = (p - q)M^{p+q}$ $[T^m, M^n] = -nM^{m+n}$
Regular M charge	finite l-Galilei	✓	(4.28)	$[P^{-l+i}, P^r] = M(-1)^{i!} \frac{(2l-i)!}{(2l)!} \delta_{-l+i+r,0}$

Table 1: Investigated central extensions for l-Galilei algebra

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References

- [1] K.G. Wilson, “Renormalization Group and Critical Phenomena. I. Renormalization Group and the Kadanoff Scaling Picture,” *Phy. Rev. B* **4**, 3174 (1971)
- [2] K.G. Wilson, “Renormalization Group and Critical Phenomena. II. Phase-Space Cell Analysis of Critical Behavior,” *Phy. Rev. B* **4**, 3184 (1971)
- [3] B. Widom, “Equation of State in the Neighborhood of the Critical Point,” *J. Chem. Phys.* **43**, 3898 (1963)
- [4] L.P. Kadanoff, “Scaling laws for Ising models near T_c ,” *Physics* **2** **263**, (1966)
- [5] A. A. Belavin, A.M. Polyakov, and A.B. Zamolodchikov “Infinite conformal symmetry in two-dimensional quantum field theory,” *Nucl. Phys. B* **241**, 333 (1984)
- [6] P. Di Francesco, P. Mathieu, and D. Senecha, “Conformal Field Theory,” (Springer, New York, 1997)
- [7] M. Henkel, “Conformal Invariance and Critical Phenomena,” (Springer, Heidelberg, 1999)
- [8] D.T. Son, “Toward an AdS/cold atoms correspondence: a geometric realization of the Schroedinger symmetry,” *Phys. Rev. D* **78**, 046003 (2008) [arXiv:0804.3972 [hep-th]].
- [9] K. Balasubramanian and J. McGreevy, “Gravity duals for non-relativistic CFTs,” *Phys. Rev. Lett.* **101**, 061601 (2008) [arXiv:0804.4053 [hep-th]].

- [10] W. Fushchych, V. Chopyk, P. Nattermann, and W. Scherer “Symmetries and reductions of nonlinear Schrödinger equations of Doebner-Goldin type,” Rep. Math. Phys. **35**, 129 (1995)
- [11] M. Henkel, M. Pleimling “Non-equilibrium phase transitions, vol 2: Ageing and dynamical scaling far from equilibrium,” Springer (Heidelberg 2010)
- [12] M. Henkel, “On logarithmic extensions of local scale-invariance,” Nucl. Phys. B **869**, 282 (2013) [arXiv:1009.4139v1 [hep-th]].
- [13] U. Niederer, “The magammamal kinematical invariance group of the free Schrödinger Helv. Phys. Acta **45**, 802 (1972).
- [14] C. R. Hagen, “Scale and conformal transformations in galilean-covariant field theory,” Phys. Rev. D **5**, 377 (1972).
- [15] P. Havas, J. Plebanski “Conformal extensions of the Galilei group and their relation to the Schrödinger group,” J. Math. Phys. **19**, 482 (1978)
- [16] M. Henkel “Local scale invariance and strongly anisotropic equilibrium critical systems,” Phys. Rev. Lett. **78**, 1940 (1997) [arXiv:cond-mat/9610174v2 [cond-mat.stat-mech]].
- [17] J. Negro, M.A. del Olmo and A. Rodríguez-Marco, *Nonrelativistic conformal groups*, J. Math. Phys. **38**, 3786 and 3810 (1997).
- [18] C. Duval, G. W. Gibbons and P. Horvathy, “Celestial Mechanics, Conformal Structures, and Gravitational Waves,” Phys. Rev. D **43**, 3907 (1991) [arXiv:hep-th/0512188].
- [19] C. Duval and P.A. Horvathy, “Non-relativistic conformal symmetries and Newton-Cartan structures,” J. Phys. A **42**, 465206 (2009) [arXiv:0904.0531 [math-ph]].
- [20] M. Henkel, R. Schott, S. Stoimenov, J. Unterberger, “The Poincare algebra in the context of ageing systems: Lie structure, representations, Appell systems and coherent states,” arXiv:0601028v1 [math-ph].
- [21] A. Hosseiny and S. Rouhani, “Affine Extension of Galilean Conformal Algebra in 2+1 Dimensions,” J. Math. Phys. **51**, 052307 (2010) [arXiv:0909.1203 [hep-th]].
- [22] R. Fareghbal and A. Naseh, “Flat-Space Energy-Momentum Tensor from BMS/GCA Correspondence,” [arXiv:1312.2109 [hep-th]].
- [23] A. Hosseiny and S. Rouhani, “Logarithmic Correlators in Non-relativistic Conformal Field Theory,” arXiv:1001.1036 [hep-th].
- [24] A. Hosseiny and A. Naseh, *On holographic realization of logarithmic Galilean conformal algebra*, J. Math. Phys. **52** 092501 (2011). [argammav:1101.2126]
- [25] J. Lukierski, P. C. Stichel and W. J. Zakrzewski, “Exotic Galilean conformal symmetry and its dynamical realisations,” Phys. Lett. A **357**, 1 (2006) [arXiv:0511259[hep-th]].

- [26] D. Grumiller, W. Riedler, J. Rosseel, T. Zojer “Holographic applications of logarithmic conformal field theories,” J. Phys. A: Math. Theor. **46**, 494002 (2013) [arXiv:1302.0280[hep-th]].
- [27] C. Duval and P. A. Horvathy, “The exotic Galilei group and the ‘Peierls substitution’,” Phys. Lett. B **479**, 284 (2000) [arXiv:0002233[hep-th]].
- [28] M. Henkel, A. Hosseiny and S. Rouhani, “Logarithmic Exotic Conformal Galilean Algebras,” Nucl. Phys. B **879**, 292 (2014) [arXiv:1311.3457[hep-th]].
- [29] C. Roger and J. Unterberger “The Schrödinger-Virasoro Lie Group and Algebra: Representation Theory and Cohomological Study” Ann. Henri Poincaré **7**, 1477 (2006)
- [30] J. Unterberger and C. Roger “The Schrödinger-Virasoro algebra,” (Springer, Heidelberg, 2011)
- [31] V. Ovsienko and C. Roger “Generalisations of Virasoro group and Virasoro algebras through extensions by modules of tensor-densities on S^1 ” Ann. Henri Poincaré **7**, 1477 (2006)
- [32] D. Giulini, “On Galilei invariance in quantum mechanics and the Bargmann superselection rule,” Ann. of Phys. **249**, 222 (1996) [arXiv:9508002[quant-ph]].
- [33] N. Aizawa and P. S. Isaac , “On irreducible representations of the exotic conformal Galilei algebra,” J. Phys. A : Math. Theor **44**, 035401 (2011) [arXiv:1010.4075 [math-ph]].
- [34] M. Henkel, “Schrödinger invariance in strongly anisotropic critical systems,” J. Statist. Phys. **75**, 1023 (1994) [arXiv:9310081[hep-th]].
- [35] A. Bagchi, R. Gopakumar, I. Mandal and A. Miwa, “GCA in 2d,” JHEP **1008**, 004 (2010) [arXiv:0912.1090 [hep-th]].
- [36] M. Pleimling and M. Henkel “Anisotropic scaling and generalized conformal invariance at Lifshitz points” Phys. Rev. Lett. **87**, 125702 (2001) [arXiv:0103194 [hep-th]].
- [37] M. Henkel “Phenomenology of local scale invariance: from conformal invariance to dynamical scaling” Nucl. Phys. B **641**, 405-486 (2002) [arXiv:0205256 [hep-th]].
- [38] D. Martelli and Y. Tachikawa, “Comments on Galilean conformal field theories and their geometric realization,” J. High. Phys. **1005**, 091 (2010) [arXiv:0903.5184 [hep-th]].